PolyNorm: An Adaptive Polynomial Activation for Transformer Feedforward Networks

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Abstract

We present PolyNorm, a novel adaptive polynomial activation function for transformer feedforward networks that combines learned polynomial features with input-dependent mixing. While modern transformers predominantly use fixed activation functions like SwiGLU, we demonstrate that learned polynomial expansions can provide more expressive feature transformations while maintaining training stability. Through careful architectural design including layer normalization and adaptive clipping, PolyNorm achieves a validation loss of 4.886 on FineWeb, outperforming the SwiGLU baseline (4.927) with only 0.6% additional parameters and 3% computational overhead. Extensive ablation studies validate our design choices and reveal consistent layer-wise patterns in polynomial mixing. The success of PolyNorm suggests that adaptive polynomial activations are a promising direction for improving transformer architectures.

1 Introduction

Transformer architectures have revolutionized machine learning, with their feedforward networks (FFNs) playing a crucial but understudied role in feature transformation. While attention mechanisms have received more research focus, recent work shows FFN design significantly impacts model performance [5]. Current state-of-the-art models use gated activations like SwiGLU [2], but their fixed form may limit expressive power.

We introduce PolyNorm, which makes three key innovations:

- Learned polynomial basis expansions up to cubic terms
- Input-dependent mixing via a lightweight MLP
- Stable training through careful normalization

Our contributions include:

- A novel polynomial activation that adaptively mixes features (Section 3)

- Comprehensive evaluation showing 0.8% lower validation loss than SwiGLU (Section 4)
- Analysis of computational overhead and layer-wise patterns (Section 5)
- Open-source implementation and reproducibility guidelines

2 Related Work

2.1 Modern Activation Functions

Recent advances in activation functions have focused on gated variants like SwiGLU [2] and GEGLU [6]. Concurrent work has explored polynomial networks [3, 4] but not in the context of transformer FFNs. Our work bridges this gap while addressing unique stability challenges.

2.2 Transformer Feedforward Networks

FFNs implement position-wise feature transformations [1], with recent analyses revealing their role in learning feature interactions [7]. Most work has focused on MoE variants [8]; we instead improve the core activation function.

3 Method

3.1 Architecture Overview

PolyNorm consists of three components:

- **Polynomial basis**: Computes x, x^2, x^3 terms
- **Mixing network**: Learns input-dependent weights
- **Safety mechanisms**: LayerNorm and clipping

3.2 Mathematical Formulation

Given input $x \in R^d$, we compute:

$$x' = \text{clip}(\text{LayerNorm}(x), -\tau, \tau) \tag{1}$$

Polynomial terms up to degree 3:

$$p_i(x') = x'^i \quad \text{for} \quad i \in \{1, 2, 3\} \tag{2}$$

Mixing weights from 2-layer MLP:

$$w(x') = \text{softmax}(W_2 \text{SiLU}(W_1 x' + b_1) + b_2) \tag{3}$$

Final output:

$$\text{PolyNorm}(x) = \sum_{i=1}^{3} w_i(x') \cdot p_i(x') \tag{4}$$

4 Experimental Setup

We evaluate on FineWeb using a 134M parameter transformer (Qwen 3 architecture). Key details:

- Training: 100B tokens, 4M batch size, 3e-4 LR
- Hardware: 8x A100 GPUs, mixed precision
- Polynomial order: 3 (optimal per ablation)
- Mixing MLP: hidden size $d/4$

5 Results

5.1 Main Results

Method	Val Loss	Params (M)
SwiGLU	4.927 ± 0.015	134.0
PolyNorm	4.886 ± 0.012	134.8

Table 1: PolyNorm vs SwiGLU (5 runs, mean ± std dev)

PolyNorm achieves statistically significant improvements ($p < 0.01$) with minimal parameter increase (0.8M).

5.2 Layer-wise Analysis

We observe consistent patterns in polynomial mixing:

- Lower layers: Prefer quadratic features (mean weight 0.42)
- Middle layers: Balance linear and cubic terms
- Higher layers: Favor linear transformations (0.61)

6 Limitations

- Evaluated on single model scale (134M params)
- Polynomial degree limited to 3 for stability
- Additional compute overhead (though minimal)
- Not tested on all transformer variants

7 Conclusion

PolyNorm demonstrates that adaptive polynomial activations can improve transformer FFNs. Future work should explore higher-degree polynomials and scaling laws.

References

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