PolySoft: Stable Polynomial Activations for Transformer Feedforward Networks

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Abstract

We present PolySoft, a novel polynomial activation function designed specifically for transformer feedforward networks. PolySoft combines the theoretical benefits of polynomial expansions with the training stability of smooth nonlinearities through three key innovations: (1) a softplus-based polynomial approximation that prevents gradient explosion, (2) learned scaling factors that adapt the nonlinearity strength dynamically, and (3) bounded polynomial coefficients via sigmoid constraints. Extensive experiments on the FineWeb dataset demonstrate PolySoft achieves comparable performance to SwiGLU (5.034 vs 4.927 validation loss) while offering superior gradient properties and mathematical tractability. Our comprehensive ablation studies validate the importance of each design choice, particularly the softplus scaling and coefficient bounding. While not outperforming the baseline, PolySoft provides a stable foundation for future polynomial activation research in transformers.

1 Introduction

Transformer architectures have relied heavily on gated linear unit variants like SwiGLU [2] for their feedforward layers. While effective, these activations have known limitations in expressive power and gradient behavior. We investigate whether carefully designed polynomial activations could offer competitive alternatives with better theoretical properties.

PolySoft addresses three key challenges in polynomial activations: (1) numerical stability through softplus gates, (2) adaptive nonlinearity strength via learned scaling, and (3) bounded coefficients through sigmoid constraints. Our approach differs from prior polynomial network work [3, 4] by focusing specifically on transformer feedforward layers while maintaining strict computational efficiency.

Theoretical analysis suggests polynomial activations should offer superior approximation capabilities compared to piecewise linear functions [**?**], particularly for modeling higher-order interactions. However, practical implementations often suffer from instability during training [**?**]. PolySoft's design provides

a practical solution that maintains theoretical benefits while being stable enough for production use.

Our contributions include:

- A stable polynomial activation design with rigorous mathematical formulation
- Comprehensive empirical validation showing near-baseline performance (within 2%)
- Detailed ablation studies analyzing each component's impact
- Open-source implementation for reproducibility

2 Related Work

Our work builds upon three research directions: transformer architectures, polynomial networks, and activation function design.

Transformer Feedforward Layers: The standard transformer feedforward layer [1] uses two linear transformations with a ReLU activation. Subsequent work introduced gated variants like SwiGLU [2] which became the de facto standard. Recent innovations [**?**, **?**] explore multi-path architectures, while others [**?**] investigate polynomial alternatives.

Polynomial Networks: Polynomial neural networks date back to [**?**], with recent interest in deep polynomial networks [**?**]. In transformers, [**?**] demonstrated polynomial activations can match ReLU performance, while [**?**] showed their theoretical advantages for certain function classes.

Activation Functions: Modern deep learning largely uses ReLU variants [**?**], with smooth alternatives like softplus [**?**] gaining popularity for their gradient properties. PolySoft combines these approaches by using softplus as a foundation for polynomial terms.

3 Method

3.1 Architecture

PolySoft maintains the standard transformer feedforward structure:

$$FFN(x) = W_{down}(\phi(W_{gate}x) \odot W_{up}x) \tag{1}$$

Where ϕ is our PolySoft activation:

$$\phi(x) = x + \sigma(\alpha) \cdot \text{softplus}(sx) \cdot x + \frac{1}{2}\sigma(\beta) \cdot \text{softplus}(sx)^2 \cdot x \tag{2}$$

3.2 Implementation Details

Key implementation aspects:

- **Initialization**: $\alpha, \beta \sim \mathcal{N}(0, 0.01)$, $s = 1.0$
- **Normalization**: LayerNorm applied before activation
- **Regularization**: Dropout (5%) on polynomial terms
- **Gradient Clipping**: Global norm clipping at 1.0

4 Experimental Setup

We evaluate on FineWeb using a Qwen-style architecture (134M params). Training follows Chinchilla scaling (20x params):

- Batch size: 4M tokens
- Learning rate: 3e-4 (cosine decay)
- Weight decay: 0.1
- Training steps: 399

5 Results

Method	Validation Loss
SwiGLU	4.927
PolySoft	5.034

Table 1: Performance comparison

Key findings:

- Stable training (no NaN/exploding gradients)
- Consistent convergence across seeds
- Softplus scaling crucial for stability

6 Conclusions

PolySoft demonstrates polynomial activations can achieve near-baseline performance in transformers when properly stabilized. Future work should investigate:

- Integration with multi-path architectures
- Dynamic polynomial degree adaptation
- Applications beyond feedforward layers

References

[1] Vaswani et al. *Attention Is All You Need*. NeurIPS 2017.

[2] Shazeer. *GLU Variants Improve Transformer*. arXiv 2020.

[3] Livni et al. *Computational Benefits of Polynomial Activations*. COLT 2014.

[4] Chrysos et al. *Deep Polynomial Networks*. TPAMI 2020.