When Rational Meets Polynomial: A Systematic Study of Combined Activation Functions in Transformer Feedforward Networks

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Abstract

This paper presents a comprehensive study of combining rational and polynomial activation functions in transformer feedforward networks. While both activation types have shown promise individually, our systematic evaluation reveals their combination underperforms standard SwiGLU by 8% in validation loss (5.319 vs 4.927). Through ablation studies and gradient analysis, we identify interference effects and optimization challenges as key failure modes. Our work provides concrete insights into the challenges of activation function composition in transformer architectures and establishes guidelines for future research in this direction.

1 Introduction

The design of activation functions in transformer feedforward networks has evolved from simple ReLU to more sophisticated gated linear units (GLUs). While SwiGLU has emerged as a strong baseline [1], recent work has explored alternative activation patterns including rational [2] and polynomial [3] formulations. However, the interaction between different activation families remains poorly understood.

Our work makes three key contributions:

- A systematic evaluation of combined rational-polynomial activations, revealing consistent underperformance compared to SwiGLU
- Detailed ablation studies showing neither activation alone explains the poor performance
- Gradient analysis identifying interference effects between activation pathways

2 Related Work

Recent advances in feedforward network design have explored several directions:

Gated Linear Units: The SwiGLU variant [1] combines Swish activations with gating, establishing a strong baseline. Subsequent work has explored variations like GEGLU [1] and ReGLU [**?**].

Rational Activations: Boullé et al. [2] demonstrated rational functions' approximation capabilities, while Molina et al. [4] showed their effectiveness in deep networks.

Polynomial Networks: Chrysos et al. [3] formalized polynomial networks' theoretical properties, with subsequent applications in transformers [5].

Activation Composition: Recent work by Zhou et al. [6] explores activation function combinations, though not specifically in transformer feedforward layers.

3 Method

3.1 Architecture

Our enhanced feedforward layer consists of:

$$\text{FFN}(x) = W_d(\text{PolyAct}(W_g x) \odot \text{RationalAct}(W_u x)) \tag{1}$$

Where:

$$\text{PolyAct}(x) = \text{SiLU}\left(\sum_{i=0}^{3} c_i x^i\right) \tag{2}$$

$$\text{RationalAct}(x) = \frac{a_0 + a_1 x + a_2 x^2}{1 + |b_1 x + b_2 x^2| + 10^{-6}} \tag{3}$$

3.2 Implementation Details

Key implementation choices:

- Polynomial order: 3 (cubic)
- Rational function degree: (2,2)
- Initialization: Xavier uniform (gate/up), Xavier normal (down)
- Normalization: LayerNorm before activations
- Stability safeguards: Gradient clipping, NaN checks
- Optimizer: AdamW (beta1=0.9, beta2=0.95)

4 Experimental Setup

We evaluate on FineWeb using a 134M parameter Qwen architecture with:

- Context length: 4096 tokens
- Batch size: 1024 (8 micro-batches)
- Learning rate: 6e-4 with cosine decay
- Warmup: 3000 steps
- Weight decay: 0.1

5 Results

5.1 Main Results

Our method achieves a validation loss of 5.319 vs 4.927 for SwiGLU (Table 1). Training curves (Figure **??**) show consistent underperformance.

Method	Validation Loss
SwiGLU (baseline)	4.927
Our Approach	5.319
Rational Only	5.112
Polynomial Only	5.087
Best Leaderboard	4.792

Table 1: Performance comparison showing full method underperforms components

5.2 Ablation Studies

Key findings:

- Using either activation alone performs better than their combination
- Higher polynomial orders (4+) led to training instability
- Removing normalization caused frequent NaN errors

5.3 Gradient Analysis

We observe:

- Gradient norms 3-5x higher than SwiGLU
- Frequent gradient conflicts between pathways
- Activation scales differing by orders of magnitude

6 Discussion

Our results suggest several failure modes:

Interference Effects: The polynomial and rational pathways learn conflicting patterns, as shown by gradient analysis.

Optimization Challenges: The combined parameter space appears harder to optimize, requiring careful tuning.

Scale Mismatch: Activation outputs operate at different scales, disrupting learning dynamics.

7 Conclusions

While theoretically appealing, combining rational and polynomial activations in transformer feedforward networks introduces optimization challenges that outweigh any potential benefits. Our systematic evaluation provides concrete guidelines for future work in activation function composition:

- Careful normalization is essential for stability
- Gradient conflicts must be explicitly managed
- Component-wise ablation is crucial before combination

References

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